[Q1]

(1) $Y_{ict} = \mu_c + \gamma_t + \alpha F_{ct} + \varepsilon_{ict}.$

(2)
$$\hat{\alpha} = (\bar{Y}_{22} - \bar{Y}_{21}) - (\bar{Y}_{12} - \bar{Y}_{11})$$
$$\hat{\alpha} \to_p \left(E[Y_{i22}] - E[Y_{i21}]\right) - \left(E[Y_{i12}] - E[Y_{i11}]\right)$$
$$= \alpha.$$

(3) If $F_{12} = F_{22} = 1$ then the "time effect" γ_2 is not separately identificable from α, the policy effect. Our rank assumption does not hold.

(4) Yes. We have:
$$E[Y_{i22}] = \mu_2 + \gamma_2 + \alpha$$
$$E[Y_{i12}] = \mu_1 + \gamma_2 + \alpha.$$
$$E[Y_{i21}] = \mu_2 + \gamma_1$$
$$E[Y_{i11}] = \mu_1 + \gamma_1 + \alpha.$$

So
$$\alpha = \left(\underbrace{E[Y_{i22}] - E[Y_{i21}]}_{= \alpha + \gamma_2 - \gamma_1}\right) - \left(\underbrace{E[Y_{i12}] - E[Y_{i11}]}_{= \gamma_2 - \gamma_1.}\right)$$

and we can estimate w/ sample means.

Q2

(1) $\hat{\alpha} = (\bar{Y}_{22} - \bar{Y}_{21}) - (\bar{Y}_{12} - \bar{Y}_{11})$.

$$\hat{V}_{\hat{\alpha}} = S^2_{22}/N_{22} + S^2_{21}/N_{21} + S^2_{12}/N_{12} + \frac{S^2_{11}}{N_{11}}$$

where S^2_{ct} is sample variance.

confidence interval: $\hat{\alpha} \pm 1.96 \times \sqrt{\hat{V}_{\hat{\alpha}}}$.

· assuming iid data over countries and time

(2)
$$\hat{\alpha} = 1/N_2 \sum \Delta Y_{i2} - 1/N_1 \sum \Delta Y_{i1}$$

$$\hat{V}_{\hat{\alpha}} = \frac{S^2_{\Delta Y,2}}{N_2} + \frac{S^2_{\Delta Y,1}}{N_1}$$

where $S^2_{\Delta Y,c}$ is sample variance of ΔY_{ic}.

CI: $\hat{\alpha} \pm 1.96 \sqrt{\hat{V}_{\hat{\alpha}}}$.

Assumptions: individuals are sampled iid.

Q3

(1) $Y_{ict} = \mu_c + \gamma_t + \alpha F_{ct} + \varepsilon_{ict}$

(2) Let $X_i = [T_i \quad C_i \quad F_i]$
be vector of time dummies, country dummies and Fracking dummies for observation i.

$$\rightarrow \quad Y_i = X_i\beta + \varepsilon_i, \quad \beta = \begin{bmatrix} \gamma \\ \mu \\ \alpha \end{bmatrix}.$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y_i$$

$\hat{\alpha}$ is last component of $\hat{\beta}$.

(3) • calculate $\hat{V}_{\hat{\beta}} = S^2_{\hat{\varepsilon}} (X^T X)^{-1}$. and take $\hat{V}_{\hat{\alpha}}$ from bottom diagonal entry of $\hat{V}_{\hat{\beta}}$.

$H_0: \alpha = 0$

$H_1: \alpha \neq 0.$

• we reject H_0 if $\left| \frac{\hat{\alpha}}{\sqrt{\hat{V}_{\hat{\alpha}}}} \right| > 1.96$

(two-sided test).

(4) $Y_{ict} = \mu_c + \gamma_t + \alpha_0 F_{ct} + \alpha_1 M_c + \varepsilon_{ict}.$

(5) Let $X_i = [T_i \quad C_i \quad F_i \quad M_i F_i]$ where M_i is a dummy for mining presence.

Model becomes:

$$Y_i = X_i\beta + \varepsilon_i, \quad \beta = \begin{bmatrix} \gamma \\ \mu \\ \alpha_0 \\ \alpha_1 \end{bmatrix}$$

• estimate $\hat{\beta} = (X^T X)^{-1} X^T Y$

and take $(\hat{\alpha}_0, \hat{\alpha}_1)$ as bottom two entries.

- estimate $\hat{V}_{\hat{\beta}} = s^2_{\hat{\varepsilon}} (X^T X)^{-1}$
- Null Hypothesis is $\alpha_1 = \alpha_0$
- write $H_0 : R\beta = 0$

  where $R = [\underbrace{0, 0, \ldots, 0}_{\text{all zeros.}}, 1, -1]$

- Assume $H_1 : R\beta \neq 0$ and reject H_0 if $\left| \dfrac{R^T \hat{\beta}}{\sqrt{R \hat{V}_{\hat{\beta}} R^T}} \right| > 1.96.$

[Q4]

(1) $Y_{ict} = \mu_c + \gamma_t + \alpha_0 F_{ct} + \alpha_1 TF_{ct} \times F_{ct} + \varepsilon_{ict}$

(2) Let $X_i = [T_i \quad C_i \quad F_i \quad F_i \times TF_i]$ for obs i.

Follow same steps as 3.4 / 3.5 for estimation of (α_0, α_1).

Here null hypothesis is $\alpha_1 = 0$.

"

Assume $H_1: \alpha_1 \neq 0$
and reject if $\left| \frac{\hat{\alpha}_1}{\sqrt{\hat{V}_{\hat{\alpha},1}}} \right| > 1.96$.

[Q5]

Parallel trends implies
$$\pi_{st} = \mu_s + \gamma_t \text{ for}$$
periods w/out treatment.

(1) $\pi_{12} - \pi_{11} = \pi_{22} - \pi_{21}$ implied by above.

(2) $\pi_{21} - \pi_{11} = \pi_{22} - \pi_{12}$ implied by above.

(3) Trick question! These hypotheses are the same. w/ iid samples, the null implies:
$$\frac{\bar{Y}_{12} - \bar{Y}_{11} - (\bar{Y}_{22} - \bar{Y}_{21})}{\sqrt{S_{11}^2/N_{11} + S_{12}^2/N_{12} + S_{21}^2/N_{21} + S_{22}^2/N_{22}}} \sim N(0,1).$$

Reject H_0 if $| \cdot | > 1.96$.

→ we are rejecting Parallel trends which invalidates our results so far.

(4) $C=3$ implies:

- $\pi_{32} - \pi_{31} = \pi_{22} - \pi_{21}$
- $\pi_{32} - \pi_{31} = \pi_{12} - \pi_{11}$.

Write all 3 restrictions as

$$R\pi = 0$$

where:

$$\pi = \begin{bmatrix} \pi_{11} \\ \pi_{12} \\ \pi_{21} \\ \pi_{22} \\ \pi_{31} \\ \pi_{32} \end{bmatrix}$$

$$R = \begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \end{bmatrix}$$

test stat is $(R\hat{\pi})^T (R\hat{V}_{\hat{\pi}} R^T)^{-1} (R\hat{\pi})$

where $\hat{\pi}$ are sample means,

$$\hat{V}_{\hat{\pi}} = \begin{bmatrix} s^2_{11}/N_{11} & & \text{zeros} \\ & \ddots & \\ \text{zeros} & & s^2_{ct}/N_{ct} \\ & & & \ddots \end{bmatrix}$$

diagonal matrix

Reject if test stat $> \chi^2_{3,0.05}$.

[Q6]

(1) Use same OLS strategy as previous questions.

w/ $P_{ict} = \mu_c + \gamma_t + \kappa F_{ct} + \varepsilon_{ict}$.

(2) Use OLS variance formula.

(3) Benefit $= \alpha + \kappa q$.

$$\hat{B} = \alpha + \hat{\kappa} q.$$

$$\mathbb{V}[\hat{B}] = q^2 \, \mathbb{V}[\hat{\kappa}].$$

so confidence interval: $\alpha + \hat{\kappa} q \pm 1.96 \times q \times \sqrt{\hat{V}_{\hat{\kappa}}}$

where $\hat{V}_{\hat{\kappa}}$ is taken from OLS variance matrix (given in previous answers).

[Q7]

(1) $\alpha_0 = E[Y_{i01}] - E[Y_{i00}]$

$\rightarrow \quad \hat{\alpha}_0 = \bar{Y}_{01} - \bar{Y}_{00}$.

(2) $\alpha_1 = E[Y_{i11}] - E[Y_{i10}] - \alpha_0$.

$$\hat{\alpha}_1 = (\bar{Y}_{11} - \bar{Y}_{10}) - (\bar{Y}_{01} - \bar{Y}_{00})$$

(3) Randomization implies $E[Y_{i10}] = E[Y_{i00}]$

so test w/ $$\left| \frac{\bar{Y}_{10} - \bar{Y}_{00}}{\sqrt{S^2_{10}/N_1 + S^2_{00}/N_0}} \right|$$

reject if $> Z_{\alpha/2}$.

(8) See your lecture notes for this question.

(9)

(1) OLS will converge

to $$\frac{C(\log(h_i), \log(w_i))}{V[\log(w_i)]} = \psi_i + \frac{C(\alpha_i, \mu_i)}{V[\log(w_i)]}$$

→ since $C(\mu_i, \alpha_i) > 0$, +ve bias.

(2) We can estimate ψ via TSLS,

using $Z_i = [x_{ic} \; v_c]$ as instruments.

X_{ic} is not strictly necessary
as long as U_c is a relevant instrument.

(3) • Null hypothesis is $\psi_1 = 0$.

• estimate $\hat{V}_{\psi}$ as $S^2_{\hat{\varepsilon}} \left(X^T Z (Z^T Z)^{-1} Z^T X \right)^{-1}$

where X is vector of random variables
used in 1st stage.

• take $\hat{V}_{\psi_1}$ as bottom right.

• Reject H_0 if $\left| \frac{\hat{\psi}_1}{\sqrt{\hat{V}_{\psi_1}}} \right| > Z_{\alpha/2}$

size.

→ assuming Homoskedasticity

and two-sided alternative.

[Q10]

(1) $\mathbb{C}(\alpha_i, \phi_c) > 0$.

(2) $E[\phi_c \mid U_{ct}] = E[\phi_c]$.

(3) Let $\alpha_i = \delta_c + \eta_i$

where $\eta_i = d_i - E[d_i \mid \text{country } c]$

model: $\log(h_{ict}) = \psi_0 + \psi \log(w_{ict}) + \delta_c + \eta_i + \varepsilon_{ict}$

$$\log(w_{ict}) = \phi_c + \gamma \varphi_c + \gamma \xi_{ct}$$

→ now we add a dummy for country in both the 1st and 2nd stage.

→ this is valid if $E[\eta_i + \varepsilon_{ict} \mid \xi_{ict}] = 0$.

[Q11]

$$H_0: \quad \pi_1 = 0, \quad \pi_2 = 0, \quad \vdots \quad \pi_L = 0.$$

$\iff \quad H_0: R\pi = 0 \quad$ where $\pi = \begin{bmatrix} \pi_0 \\ \pi_1 \\ \vdots \\ \pi_L \end{bmatrix}$

$$R = \begin{bmatrix} 0 & — & 0 & & \\ 0 & 1 & 0 & — & \text{zeros} \\ 0 & 0 & 1 & — & \\ & & & 1 & \\ & \text{zeros} & & & \ddots \; 1 \end{bmatrix}$$

ones in all but 1st diagonal entry.

test stat: $(R\hat{\Pi})^T (R^T \hat{V}_{\hat{\Pi}} R)^{-1} (R\hat{\Pi})$

where $\hat{\Pi}$ is OLS estimator,

$\hat{V}_{\hat{\Pi}}$ is variance estimate. (e.g. $s^2_{\hat{\varepsilon}}(Z'Z)^{-1}$).

Reject H_0 if test stat $> \chi^2_{L,\alpha}$

chosen size

[Q12]

(1) This would violate the rank assumption on the instruments

(i.e. the relevance of instruments assumption)

(2) Estimate the model

$$X_i = \Pi_0 + \Pi_1 Z_{1i} + \Pi_2 Z_{2i} + \varepsilon_i$$

and conduct a joint hypothesis test

H_0: $\Pi_1 = 0$

$\Pi_2 = 0$.

(using Wald statistic)

(3) If Z_1 & Z_2 make collinear

more rewarding / less costly, this increases the returns to graduating high school.

[Q13]

(1) $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$x_i^m = x_i + \eta_i$$

$\rightarrow$ $Y_i = \beta_0 + \beta_1 (x_i^m - \eta_i) + \varepsilon_i$

$$= \beta_0 + \beta_1 x_i^m - \beta_1 \eta_i + \varepsilon_i.$$

$\Rightarrow$ note that $E[\eta_i \mid x_i^m] \neq 0$ since $x_i^m = x_i + \eta_i$ and strict exogeneity violated.

- alternatively, OLS converges to:

$$\frac{C(Y_i, x_i^m)}{V[x_i^m]} = \frac{C(\beta_0 + \beta_1 x_i, x_i + \eta_i)}{V[x_i] + \sigma_\eta^2}$$

$$= \beta_1 \times \left(\frac{V[x_i]}{V[x_i] + \sigma_\eta^2} \right) \neq \beta_1$$

(2) We can now estimate β_1 by 2SLS using z_i as an instrument for x_i^m, since: $E[\eta_i \mid z_i] = 0$.

Alternatively, 2SLS converges to: $\dfrac{C(Y_i, z_i)}{C(x_i^m, z_i)}$

$$= \frac{C(\beta_0 + \beta_1 x_i + \varepsilon_i,\ x_i + \zeta_i)}{C(x_i + \eta_i,\ x_i + \zeta_i)} = \beta_1 \times \frac{\cancel{V[x_i]}}{\cancel{V[x_i]}} \quad //.$$